# Appendices:

# Appendix A: Proof of Proposition 1

Before depicting the proof of Proposition 1, we introduce the following notation. Let denote firm quasi-rents in period t, and let subscript *j* denote early adopters (i.e., *j=1*), late adopters (i.e., *j=2*), or units that do not adopt (i.e., *j=0*).

We use this notation to define the first- and second-period survival regions:

I) The *first-period survival region* is

where , and the following curves define :

1) (i.e., point D in Figure 1):

2) (i.e., line EP in Figure 1):

3) (i.e., line SD in Figure 1):

4) (i.e., line VU in Figure 1):

Note that the set defines the line EBCD in Figure 1.

II) The *second-period survival region* is

where , and the values are defined as follows:

1)

2)

3)

4)

These definitions suggest that and . Because policy affects both first-period and second-period survival regions, it affects not only , and , but also , and for j=A,B.

We use the aforementioned survival regions to characterize the optimal solution to Party A, while beginning with period 2.

Party A chooses to maximize Eq. (6) and sets (i.e., the static second-period Pigovian tax).

Party A chooses to maximize Eq. (5) and the First Order Coditions (F.O.C.) of this maximization problem is

In deriving the F.O.C., we use the chain rule and employ the envelope theorem and thus

(follows from the F.O.C. of the second period assuming Party A remains in power). Furthermore, because investment is irreversible and , the optimal first-period tax per pollution unit is

The final step of the proof is to show that the optimal tariff scheme derived above yields, in equilibrium, the solution that maximizes Party A’s objective function. To derive this conclusion, recall that a firm is active and produces at capacity if its profit is non-negative but becomes idle otherwise, and that is a smooth function with compact support. Furthermore, assuming that policy is binding suggests that the marginal unit earns zero profits; that is, the marginal unit equates its benefit from producing one unit with the cost of the pollution it creates. Let for denote the policy in equilibrium in period *t* and assume and . By construction, marginal firms under the optimal tariff scheme are the marginal units in the equilibium and vis versa. Because firms’ expected profits decline with and , the tax rates, and , maximize .

The proposition follows.

Q.E.D.

**Appendix B: Proof of Proposition 3**

## B1. The tax regime

The current section focuses on a tax, , and Eq. (1) is modified accordingly. Furthermore, because firms are forward looking, not only current period profits but also expected future profits affect units’ decisions.

Firms decide whether to invest in the alternative technology in the first period (i.e., early adopters), second period (i.e., late adopters), or never. This decision is made simultaneously with the decision regarding whether to remain active or become idle. These decisions try to mimic, for instance, the response of U.S. power plants to the U.S. Environmental Protection Agency’s proposal to limit emissions of coal-fired plants to 1,100 pounds of CO2 per megawatt hour. If such proposed regulation is enacted, then coal-fired plants would either invest in cleaner technologies (e.g., natural-gas fired boilers, carbon storage and capture technologies, co-generation with biomass) or retire the plant. On the other hand, natural gas-fired plants continue to produce electricity using their existing technologies.

Formally, and given policy, let denote the quasi-rents of units that modified the technology in the first period (i.e., early adopters):

|  |  |  |
| --- | --- | --- |
|  |  | (1b) |

Let denote the quasi-rents of units that adopted the technology only in the second period (i.e., late adopters):

|  |  |  |
| --- | --- | --- |
|  |  | (2b) |

Finally, let denote active units that never adopted the technology:

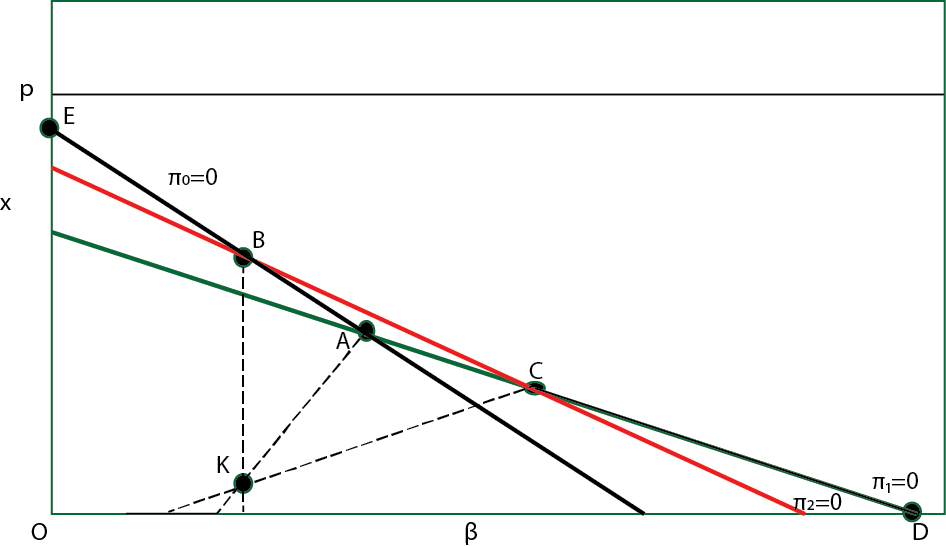
|  |  |  |
| --- | --- | --- |
|  |  | (3b) |

Next, the survival region is defined. The first-period survival region (i.e., ) depicts the region of firms that earn non-negative quasi-rents; that is, the region where or or . In other words,

**Lemma 1b.**Let for denote the line where . Then,

Furthermore, *.*

Building on Lemma 1c, we plot for in Figure 1b. These curves bound the survival region from above – that is, the survival region is region DCBEO in Figure 1c. Note that while the borders of the survival region include points B and C, point A is not included. Units at point A earn strictly positive profits; point A is down and to the left of curve .



*Figure 1b*. First period survival region under a tax

Policy resulted in inefficient and dirty units exiting the industry (i.e., units located in region ). Next, we characterized the early adopters and separated them from late adopters. We also identified units that did not adopt the technology at all.

Using Eqs. (1b) and (3b), we derived the linear relationship between input-output and the pollution-output coefficients of firms that are indifferent between adopting the modification in period 1 or not adopting at all (i.e., the line at which ). This relationship is depicted in Eq. (4b):

|  |  |  |
| --- | --- | --- |
|  |  | (4b) |

Let denote the point where (point A in Figure 1b).

Using Eqs. (1b) and (2b), we derived the linear relationship between input-output and the pollution-output coefficients of firms that are indifferent to the period they adopt the modification (i.e., the line at which ). This relationship is depicted in Eq. (5b):

|  |  |  |
| --- | --- | --- |
|  |  | (5b) |

Let denote the point where (point C in Figure 1b).

Finally, using Eqs. (2b) and (3b), we derived the relationship between input-output and the pollution-output coefficients of firms that are indifferent between adopting the modified technology only in period 2 or not adopting the modification (i.e., the line at which ). This relationship is summarized by Eq. (6b):

|  |  |  |
| --- | --- | --- |
|  |  | (6b) |

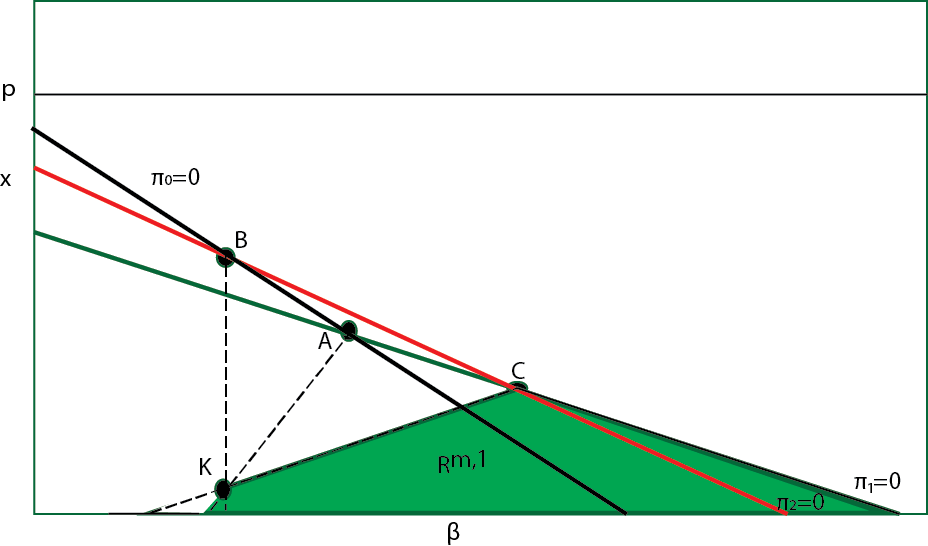
Let denote the point where (point B in Figure 1b). Note that the indifference between adopting the modified technology only in period 2 and not adopting the modification at all is independent of (Eq. (6b)). This result is the outcome of our assumption that adopting the modified technology in the second period does not affect production costs.

Using Eqs. (4b) and (5b), we can also show the following:

**Lemma 2b.** If then .

Lemma 2b tells us that when Eqs. (4b), (5b), and (6b) intersect, (Figure 1b, point K).

Let denote the modification region whereby units located in this region modified their technology in the first period. We depicted this region in Figure 2b. Firms located in region – the region in green in Figure 2b – earn highest quasi-rents when adopting the pollution abatement technology in the first period. The units that modify the technology in the first period are the relative dirty but efficient units. The model predicts that if the proposed U.S. Environmental Protection Agency’s proposal for regulating coal-fired power plants is enacted, it will lead to inefficient coal plants shutting down, but the relatively efficient plants will shift to more environmentally benign technologies.



*Figure 2b*. Survival region and the early adopters under a tax